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Number Systems and Binary Codes

1.1 DIGITAL ELECTRONICS

Digital devices are based on electronic circuitry, which represent two states viz. ON or 1 (high) state and OFF or 0 (low) state. The ability to design integrated circuits consisting of several transistors, diodes, resistors,etc. on a thin wafer — small sized chip has made digital electronics a very feasible and potent technology. Digital circuits are used to store, evaluate and modify digital codes. Integrated circuits have made it possible to design circuits which are capable of handling large quantities of digital encoded data at enormously high speeds. The word digital means discrete units like the number of fingers, match sticks etc., which can express a whole number. Analog means establishing similarities between two quantities. Analog numbers are thus represented by directly measurable quantities such as voltage, current, resistance, etc. Examples of analog devices are voltmeter, ammeter, slide rule, speedometer, etc. Both the systems — digital and analog are used in control systems, instrumentation, communications, computers and industrial automation. Digital methods of operation offer greater speed precision, accuracy and are more reliable and effective than analog methods. Digital methods are also less affected by noise as compared to analog methods. Moreover the information can be easily stored in digital systems.

In digital systems the terms *bit*, *nibble* and *byte* are quite frequently used. **Bit** is an abbreviated form of **B**inary dig**IT**. Bit is the basic unit of memory and the two binary digits are 0 and 1. Any binary number can be represented by a string of these two binary digits. **Byte** is a string of 8 bits such as 10011011, 01010111 etc. It is the basic unit of binary information and storage. Most computers process data with a length of 8 bits or some multiple of 8 bits. **Nibble** is a string of 4 bits such as 1101, 1010, etc. It is half a byte. In digital systems a group of bits which is stored, operated and moved around is called a **Word**. One word consists of 16 bits or two bytes, for example 1 word is 01101001, 01011011 in which the first 8 bits represent upper byte and last 8 bits represent the longer byte. Chunking means replacing a longer string of data by a shorter one. Register is a group of electronic, megnetic or mechanical devices which stores digital data. A **program** is a sequence of instructions that tells the computer, how to process the data. It is also known as **software**. The combination of electronic, magnetic, and mechanical devices of a computer is called **hardware**. Hybrid systems are those which employ both digital and analog techniques within the same system.

1.2 INTEGRATED CIRCUITS OR CHIP

Digital circuits are constructed using integrated circuits abbreviated as IC, which is a very small silicon semiconductor crystal. It is thin like a wafer of postage stamp size and called a chip. It contains a large number of transistor, diodes, resistors, capacitors which all form an integral part of the chip and are interconnected to form an electronic circuit. The chip is mostly mounted on a plastic package and connections are brought out to external pins. Integrated circuits are available in two types of packages — the flat package and the Dual-In-Line package, the latter being most widely used due to its low price and easy installation on its base or on printed circuit boards. The IC package envelope is made of ceramic or plastic and come in standard sizes, of 8 to 64 pins. An 8085 Intel microprocessor is enclosed within a 40 pin DIP **(dual-in line package)** with dimensions 50×15×4 mm. Each manufacturer makes available a data book providing the necessary information pretaining to the number of ICs printed on the surface of package.

Digital circuits are invariably constructed using ICs due to the advantages of reduction in size, low cost, reduced power consumption, higher operating speed, higher reliability against failure thus less prone to repairs. Small Scale Integration (SSI) devices are those which have upto 10 logic gates in a single chip. An IC having 10 to 100 gates per chip is classified as Medium Scale Integration (MSI). A Large Scale Integration device (LSI) contains 100 to 10000 gates per chip. Very large Scale Integration (VLSI) devices perform logic functions with more than ten thousand gates but less than 100000 gates per chip. Utra Large Scale Integration (ULSI) devices performs logic functions and integration with 100000 or more gates per chip.

1.3 DECIMAL SYSTEM

In our ordinary system of numbers called arabic numerals, we have 10 symbols 0,1,2,...9. A number 192 pronounced as One hundred ninety two, actually means 1×100 + 9×10 + 1×1. The value of each digit is thus determined by its position. Basically 10 numerals are used, irrespective of the size of the number to be written. The decimal system has a base of 10. The base or radix of a number system is the number of different digits which can occupy each position of the system.

1.4 BINARY SYSTEM

Binary numbers have a base or radix of 2 because only two digits 0 and 1 are used. Any decimal number can be represented in the binary system by a string of 0's and 1's. In every number system, each position of the digit has a weight or value assigned to it.

Decimal to binary conversion: In the most popular method called the dibble - dabble method, the given decimal number is successively divided by 2, giving a succession of remainders of 0 or 1. The remainders read in the reverse order give the binary equivalent to the given decimal number.

EXAMPLE: (a) Convert decimal 1993 to a binary number.

2	1993	
2	996	1 (First Remainder)
2	498	0
2	249	0
2	124	1
2	62	0
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

Read in Reverse order ↑

First division by 2 yields quotient 996 and remainder 1. Next division of 996 yields quotient 498 and remainder 0. Next division of 498 yields quotient 249 and remainder 0 and so on.

Read in reverse order from bottom to top $(1993) = (11111001001)_2$ Ans.

EXAMPLE: (b) Convert Binary 11111001001 to decimal. The right most digit is called

Least significant bit LSB and has weight = $(\text{base})^0 = 2^0$

The second digit from right has weight 2^1, third digit from right has weight 2^2 and so on.

Solution:

$\therefore$ (11111001001)

$= 1\times2^{10} + 1\times2^9 + 1\times2^8 + 1\times2^7 + 1\times2^6 + 0\times2^5 + 0\times2^4 + 1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$

$= 1024 + 512 + 256 + 128 + 64 + 0 + 0 + 8 + 0 + 0 + 1$

$= 1993$ Ans.

EXAMPLE 2: Convert Binary 11.0111 to decimal.

Solution:

The first digit after decimal point on the right has weight 2^{-1}, the second digit has 2^{-2}, the third digit has 2^{-3} and so on. First digit before decimal point has weight 2^0, the second has 2^1 and so on.

$\therefore$ 11.0111

$= 1\times2^1 + 1\times2^0 + 0\times2^{-1} + 1\times2^{-2} + 1\times2^{-3} + 1\times2^{-4}$

$= 2 + 1 + 0 + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 3\frac{7}{16}$ Ans.

EXAMPLE 3: Convert binary 1100.11 to decimal

Solution:

$\therefore$ 1100.11

$= 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-2}$

$= 8 + 4 + 0 + 0 + \frac{1}{2} + \frac{1}{4} = 12\frac{3}{4} = 12.75$ Ans.

EXAMPLE 4: Convert $\frac{27}{32}$ decimal to binary

$$\frac{27}{32} = \frac{16}{32} + \frac{8}{32} + \frac{2}{32} + \frac{1}{32}$$

$$= 1 \times \frac{16}{32} + 1 \times \frac{8}{32} + 0 \times \frac{4}{32} + 1 \times \frac{2}{32} + 1 \times \frac{1}{32}$$

$= 0.11.11$ Ans.

1.5 OCTAL SYSTEM

The octal number system has a base or radix of 8. The eight symbols used are 0,1,2,3,4,5,6,7. Decimal to octal conversion can be done by the dibble-dabble method by successively dividing by 8, giving succession of remainders lying between 0 and 7. The remainders written in reverse order gives the octal equivalent of given decimal number.

EXAMPLE 5: Convert decimal 1993 to octal.

8 | 1993	1	First remainder
8 | 249	1 ↑	
8 | 31	7	Reverse
0	3	order

Solution:
First division of 1993 by 8 yield, quotient 249 and remainder 1. The next division of 249 by 8 yields quotient 31 and remainder 1 and so on.
$\therefore$ (1993) = (3711) Ans.

EXAMPLE 6: Convert 3711 octal to decimal. The right most digit has weight 8^0, the second digit from right has weight 8^1 and so on.

Solution:

$$\therefore (3711)_8 = 3\times8^3 + 7\times8^2 + 1\times8^1 + 1\times8^0$$
$$= 3\times512 + 7\times64 + 1\times8 + 1\times1$$
$$= 1536 + 448 + 8 + 1 = 1993 \text{ Ans.}$$

The conversion of a binary number to an octal number can be done by separating the binary number into groups of 3 bits starting from right and writing decimal equivalents of each group.

EXAMPLE 7: Convert 11 111 001 001 binary to octal.

Solution:
Octal 3711 can again be written back in binary by writing 3 digit binary equivalents of each octal digit.

3 7 1 1

011 111 001 001 = 11 111 001 001 Ans.

EXAMPLE 8 (a): Convert decimal 1996 to an equivalent binary number.

Solution:

We will use here the dibble-dabble method of successively dividing the given decimal number by 2.

		Remainder	
2	1996		
2	998	0	→ First remainder
2	499	0	
2	249	1	
2	124	1	
2	62	0	
2	31	0	
2	15	1	↑ Read in Reverse order
2	7	1	
2	3	1	
2	1	1	
	0	1	

The first division of 1996 by 2 yields a quotient 998 and remainder 0. The next division of 998 by 2 yields a quotient 499 and remainder 0. The next division of 499 by 2 yields a quotient 249 and a remainder 1 and so on. When these remainders are ready in reverse order from bottom to top, they yield a binary number 11111001100, which is the required binary equivalent of the decimal number 1996.

EXAMPLE 8(b): Convert the binary number 11111001100 to its equivalent decimal number.

Solution:

We will use here the dibble-dabble method, in which each binary digit is multiplied by its weight. Then all these products are added to yield the required decimal number. The right most digit is called LSB and has weight = $(\text{base})^0 = 2^0$.

The second digit from right has weight 2^1, the third digit from right has weight 2^2 and so on.

$\therefore (11111001100)_2$

$= 1\times2^{10} + 1\times2^9 + 1\times2^8 + 1\times2^7 + 1\times2^6 + 0\times2^5 + 0\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 0\times2^0$

$= 1024 + 512 + 256 + 128 + 64 + 0 + 0 + 8 + 4 + 0 + 0$

= 1996 Ans.

EXAMPLE 9(a)

Convert $(274.1875)_{10}$ to binary [AMIE Summer 1993 — Pulse & Digital Ckts]

(b) Convert $(0.65625)_{10}$ to binary [AMIE Summer 1993 — Pulse & Digital Ckts]

(c) Convert $(624)_{10}$ to binary [AMIE Winter 1993]

(d) A person represents himself as being $(11010)_2$ years old. What is his actual age? [AMIE Winter 1993]

Solution

(a) First we will convert the whole number part i.e., 274 by successively dividing this number by 2 as before. Then the remaining fractional part, which is after the decimal point i.e., (0.1875) is successively multiplied by 2 and the carry in each step in noted. The carry part of each step is then read in forward order.

2	274	
2	137	0 → First remainder
2	68	1
2	34	0
2	17	0
2	8	1 ↑ Read in reverse order
2	4	0
2	2	0
2	1	0
	0	1

The remainders read in reverse order yield the binary equivalent of $(274)_{10}$ = 100010010.

To get the binary equivalent of 0.1875, we successively multiply it by 2 and obtain carry of each product.

0.1875 × 2 = 0.375 with carry 0.

0.375 × 2 = 0.750 with carry 0.

0.750 × 2 = 1.500 = 0.500 with carry 1.

0.500 × 2 = 1.000 with carry 1.

Carry of each product read in forward order yields the binary equivalent of $(0.1875)_{10}$ = 0.0011.

Thus the binary equivalent of $(274.1875)_{10}$

= 100010010.0011 Ans.

(b) As the decimal number 0.65625 has only a fractional part, therefore we multiply it by 2 to obtain the carry and again multiply the resulting fractional part by 2 to obtain carry and so on.

0.65625 × 2 = 1.31125 = 0.31125 with carry 1.

0.31125 × 2 = 0.6250 with carry 0.

0.6250 × 2 = 1.250 = 0.250 with carry 1.

0.250 × 2 = 0.500 with carry 0.

0.500 × 2 = 1.000 = 0.000 with carry 1.

Therefore the binary equivalent of $(0.65625)_{10}$ is

= 0.10101 Ans.

(c) The decimal number 624 has only the whole number part and no fractional part. Hence 624 is successively divided by 2

2	624		
2	312	0	First remainder
2	156	0	
2	78	0	
2	39	0	
2	19	1	
2	9	1	↑ Read in reverse order
2	4	1	
2	2	0	
2	1	0	
	0	1	

The remainders read in the reverse order yield the binary equivalent of $(624)_{10}$ as = 1001110000 Ans.

(d) 11010

$= 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$

$= 16 + 8 + 0 + 2 + 0 = 26$ years Ans.

1.6 HEXADECIMAL SYSTEM

Hexadecimal numbers are used in microprocessor work for display and writing, as they are much shorter than their binary equivalents. They have base or radix of 16. The table 1.1 shows equivalent Hexadecimal, binary and decimal digits.

Table 1.1 Hexadecimal Number System

Hexadecimal	*Binary*	*Decimal*	*Octal*
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	8	10
9	1001	9	11
A	1010	10	12
B	1011	11	13
C	1100	12	14
D	1101	13	15
E	1110	14	16
F	1111	15	17

Decimal to hexadecimal conversion can be done by the dibble-dabble method of successively dividing by 16, giving succession of remainders lying between 0 and 15 which are re-named in hex using Table 1.1. The remainders written in reverse order gives the hex equivalent of given decimal number

16 | 1993			
16 | 124	9 First remainder	9	↑ Reverse order
16 | 7	12	C	
0	7	7	

$\therefore (1993)_{10} = (7C9)_{16}$

EXAMPLE 10 (a): Convert 7C9 hex to decimal

Solution:

$$7\,C\,9$$
$$= 7 \times 16^2 + 12 \times 16^1 + 9 \times 16^0 = 7 \times 256 + 12 \times 16 + 9 \times 1$$
$$= 1792 + 192 + 9 = 1993 \text{ Ans.}$$

EXAMPLE 10 (b): Convert decimal 2047 to hex

Solution:

16 | 2047			
16 | 127	15 First remainder	F	↑ Reverse order
16 | 7	15	F	
0	7	7	

$\therefore (2047)_{10} = (7FF)_{16}$ Ans.

Hexadecimal to binary conversion is done by writing 4-bit binary equivalent of each hex digit.

EXAMPLE 11: Convert $(CD42)_{16}$ to binary and decimal systems

Solution:

$$\begin{array}{cccc} C & D & 4 & 2 \end{array}$$
$$= 1100 \quad 1101 \quad 0100 \quad 0010 = (1100\,1101\,0100\,0010)_2$$
$$C\,D\,4\,2$$
$$= 12 \times 16^3 + 13 \times 16^2 + 4 \times 16^1 + 2 \times 16^0$$
$$= 12 \times 4096 + 13 \times 256 + 64 + 2 = 49152 + 3328 + 64 + 2$$
$$= (52546)_{10} \text{ Ans.}$$

EXAMPLE 12: Convert 100 hex to binary, octal and decimal systems

Solution:

$$\begin{array}{ccc} 1 & 0 & 0 \end{array}$$
$$= 0001 \quad 0000 \quad 0000 = (10000\,0000)_2$$
$$(100)_{16} = (100\,000\,000)_2$$
$$= (4 \quad 0 \quad 0)_8 = (400)_8$$
$$(100)_{16} = 1 \times 16^2 + 0 \times 16^1 + 0 \times 16^0$$
$$= 256 + 0 + 0 = (256)_{10} \text{ Ans.}$$